O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Fall 2002
Final Exam

Choose any four out of five problems.
Please specify which four listed below to be graded:
1)_____; 2)_____; 3)_____; 4)_____;

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________

Problem 1:

Consider the system

$$x(k+1) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u(k),$$

$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(k)$$

and let $x(0) = 0$ and $u(k) = 1,\ n \geq 0$.

a) Determine $\{y(k)\},\ k \geq 0$ by any approach.

b) If it is known that when $u(k) = 0$, then $y(0) = y(1) = 1$, can $x(0)$ be uniquely determined? If your answer is affirmative, determine $x(0)$.

Problem 2:

Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix},$$

Find e^{At} and $\sin At$

Problem 3:

Show that there exists a similarity transformation matrix P such that

$$PAP^{-1} = A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n-1} \end{bmatrix},$$

if and only if there exists a vector $b \in \Re^n$ such that the rank of $\begin{bmatrix} b & Ab & \cdots & A^{n-1}b \end{bmatrix}$ is n.

Problem 4:
Consider the matrix

$$A = \begin{bmatrix} -\alpha_1 & -\alpha_2 & -\alpha_3 & -\alpha_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

show that its characteristic polynomial is given by

$$\Delta(\lambda) = \lambda^4 + \alpha_1\lambda^3 + \alpha_2\lambda^2 + \alpha_3\lambda + \alpha_4 .$$

Show also that if λ_i is an eigenvalue of A, then $\begin{bmatrix}\lambda_i^3 & \lambda_i^2 & \lambda_i & 1\end{bmatrix}^T$ is an eigenvector pf A associated with λ_i .

Problem 5:
Consider the system representations given by

$$x(k+1) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \end{bmatrix} u(k)$$

and

$$\tilde{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k).$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0 & 1 \end{bmatrix} u(k)$$

Are these representations equivalent? Are they zero-input equivalent?